

Light emitting single electron transistors

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Abstract

The dynamic properties of light-emitting single-electron transistors (LESETs) made from quantum dots are theoretically studied by using nonequilibrium Green's function method. Holes residing at QD created by small ac signals added in the base electrode of valence band lead to the exciton assisted tunnelling level for the electron tunnelling from emitter to collector, it is therefore such small signals can be amplified. LESETs can be employed as efficient single-photon detectors.

I. INTRODUCTION

The single electron tunnelling devices made from quantum dots have been extensively studied not only for interesting physics such as Kondo effect¹ and Coulomb Blockade², but also for their promising applications in single-electron transistors (SETs)³ and single-photon generators (SPGs).⁴ SPGs provide the antibunching single-photon sources (SPSs) for being applied in quantum communications. Not only do SPSs crucial, but also single-photon detectors in the implementation of quantum communication systems. Blakesley et al have demonstrated resonant diodes as efficient single-photon detectors⁵. Nevertheless, tunnelling current through the quantum wells are not sensitive enough to distinguish the intensity of SPS in the ns period.⁵ To detect single-photon train in a short period, we proposal to employ SETs to detect such a single-photon train.

The studied SET is shown in Fig. 1, where a single quantum dot (InAs) embedded in the GaAs matrix connected with three terminals (emitter, collector and base). We consider only the ground states of conduction and valence band of the QD, which are above the Fermi energy levels of three electrodes $E_{F,i=e,c,b}$. Conventional SETs are consisted of source, drain and gate electrodes, where the gate electrode is used to tune the energy levels of QDs, but not supplies carriers. In contrast, the base terminal will provide holes into the QD in this study. In the absence of holes, the applied voltage crossing the emitter and collector is insufficient to yield significant current. Once holes residing at the QD creating new channels for electrons tunnelling from emitter to collector, the remarkable collector current (I_c) is yielded. In addition, the base current (I_b) arises from the photon emission via electron-hole recombination of the exciton complex state will be observed.

Unlike traditional transistors (electric output) and light emitting diodes (optical output), the light-emitting single-electron transistors (LESETs) with electric and optical outputs can readily reach high current gain ($\beta = I_c/I_b \gg 1$) (only voltage gain in conventional SETs). It is expected that $\beta \gg 1$ is a manifested demonstration of small signal amplifier. Therefore, it could be applied to detect single-photon train. For a small semiconductor QD, particle collection significantly influences the transport and optical properties of single electron tunnelling devices. Within the framework of the effective mass model⁶, Fig. 2 show the electron-electron interaction U_e , hole-hole interaction U_h and electron-hole

interaction U_{eh} for different exciton complex configurations. The electron-hole Coulomb energy U_{eh} in the exciton X is almost the same as that in X^2 and is omitted in this plot. Note that the magnitude of charging energies (U_e and U_h) for electrons or holes in the same order of thermal energy of room temperature $k_B T$, where k_B is a Boltzmann constant. This indicates that the operation temperature of system should be much lower than room temperature.

Owing to the applied bias crossing the QD, the electric field effect is not negligible in the variation of particle interactions. We adopt the size of QD with radius 7.5 nm and height 3 nm to study the electric field effect on the particle Coulomb interactions. We assume that the z axis is directed from the base to the apex of the dot. Fig. 3 shows the Coulomb interaction strengths as functions of electric field for different exciton complexes. We see that U_h , U_{eh} and U_e display asymmetric behavior of electric field as a result of geometry of the dots. Increasing electric field, the deduction of U_{eh} indicates that the electron-hole separation increases. However, we note that U_h increases in the positive direction of electric field since the wave functions of holes become more localized. Even though the enhancement of U_e is observed in the negative direction of electric field, it only exists at very small electric field region. When the electric field is larger than a threshold value, the wave functions of electrons become delocalized and leak out the quantum dot. Consequently, electron-electron Coulomb interactions become weak. As mentioned, U_e and U_h denote the charging energies of QD for electrons and holes, respectively. Therefore, the constant interaction model used in the Anderson model is valid only for small electric field case, otherwise we should take into account bias-dependent Coulomb interactions

II. FORMALISM

An Anderson model with two energy levels and constant interactions is used to describe the system as shown in Fig. 1,

$$\begin{aligned}
H = & \sum_k \epsilon_{k,e} a_k^\dagger a_k + \sum_k \epsilon_{k,c} b_k^\dagger b_k + \sum_k \epsilon_{k,b} c_k^\dagger c_k \\
& + \sum_{i=1,2} \epsilon_i d_i^\dagger d_i + \lambda_{12} d_1^\dagger d_2 + \lambda_{21}^* d_2^\dagger d_1
\end{aligned} \tag{1}$$

$$+ \sum_{k,1} t_{k,2} a_k^\dagger d_2 + \sum_k t_k^\dagger b_k^\dagger d_2 + \sum_k t_{k,1}^\dagger c_k^\dagger d_1 + h.c$$

where $a_k^\dagger(a_k)$, $b_k^\dagger(b_k)$ and $c_k^\dagger(c_k)$ create (destroy) an electron of momentum k in the emitter, collector and base electrodes, respectively. The free electron model is considered in the electrodes in which electrons have frequency-dependent energies $\epsilon_{k,e(c)} = \varepsilon_k - \omega/2$ and $\epsilon_{k,b} = \varepsilon_k + \omega/2 + v(t)$. Time-dependent modulation $v(t)$ denotes the time-dependent applied voltage in the base electrode. d_i^\dagger (d_i) creates (destroys) an electron inside the QD with orbital energy $\epsilon_i = E_i - (-1)^i \omega/2$. In this study $i = 1$ and $i = 2$ represent, respectively, the ground states of valence band and conduction band of individual QDs. The fifth term describes the coupling of the QD with electromagnetic field of frequency ω . $\lambda = -\mu_r \mathcal{E}$ is the Rabi frequency, where $\mu_r = \langle f|r|i \rangle$ is the matrix element for the optical transition and \mathcal{E} is the electric field per photon. $t_{k,i}$ describes the coupling between the band states of electrodes and energy levels of QD. Note that a unitary transformation, $S(t) = \exp^{i\omega t/2(\sum_k (c_k^\dagger c_k - a_k^\dagger a_k - b_k^\dagger b_k) + d_1^\dagger d_1 - d_2^\dagger d_2)}$, has been used to obtain Eq. (1) via

$$H = S^{-1} H(t) S - i S^{-1} \frac{\partial}{\partial t} S.$$

To investigate the exciton assistant process, the interlevel Coulomb interaction U_{12} (U_{eh}) is taken into account in Eq. (1)

$$H_U = U_{12} d_1^\dagger d_1 d_2^\dagger d_2, \quad (2)$$

which is invariant under unitary transformation. Because we restrict in the regime of applied voltage not sufficient to overcome the charging energies resulting from U_{ee} and U_{hh} , therefore, we ignore U_{ee} and U_{hh} terms in this study.³

The emitter current can be calculated using the nonequilibrium Keldysh Green's functions, which can be found in refs.[7,8]. Wang et al have pointed out that the displacement current arising from an applied voltage is crucial to maintain gauge invariance, which will be satisfied when the condition of total charge conservation is satisfied.⁹ The time-dependent emitter current is consisted of the collector current and base current $J_e(t) = J_c(t) + J_b(t)$. The collector current satisfying the charge conservation and gauge invariance is given by

$$J_c(t) = \frac{e}{\hbar} \frac{\Gamma_e \Gamma_c}{\Gamma_e + \Gamma_c} \int \frac{d\epsilon}{\pi} [f_e(\epsilon) - f_c(\epsilon)] \times \left[-\text{Im} A_e^r(\epsilon, t) - \frac{1}{2} \frac{d|A_e^r(\epsilon, t)|^2}{dt} \right], \quad (3)$$

where $A_e^{r(a)}(\epsilon, t) = \int dt_1 e^{\pm i\epsilon(t-t_1)} G_e^{r(a)}(t, t_1)$. $G_e^{r(a)}(t, t_1)$ denotes the retarded (advanced) Green's function of electrons. In Eq. (3) we assume that tunnelling rates $\Gamma_{e(c)} = \sum_k \delta(\epsilon - \epsilon_k)$ are energy- and bias independent. To solve the spectral function of electrons $A_e^r(\epsilon, t)$, the retarded Green's function of electrons $G_e^r(t, t_1)$ is derived to obtain

$$G_e^r(t, t_1) = (1 - N_h(t_1)) g_e^r(E_e, t, t_1) + N_h(t_1) g_e^r(E_e - U_{eh}, t, t_1) \quad (4)$$

with

$$g_e^r(E_e, t, t_1) = -i\theta(t - t_1) e^{-i(E_e - i\Gamma_e/2)(t-t_1)}, \quad (5)$$

Two branches exist in Eq. (4); one corresponds to the electron resonant energy level of E_e with a weight of $(1 - N_h(t_1))$, and the other corresponds to the exciton resonant level of $E_{ex} = E_e - U_{eh}$ with a weight of $N_h(t_1)$. Consequently, electrons injected into the energy levels of QDs depends on not only the emitter and collect voltages, but also on the hole occupation number $N_h(t)$, which is given by

$$N_h(t) = \int \frac{d\epsilon}{\pi} \Gamma_h |A_h^r(\epsilon, t)|^2 \quad (6)$$

where $A_h^{r(a)}(\epsilon, t) = \int dt_1 e^{\pm i\epsilon(t-t_1)} G_h^{r(a)}(t, t_1)$. The retarded Green's function of holes is given by

$$G_h^r(E_h, t, t_1) = -i\theta(t - t_1) e^{-i(E_h - i\Gamma_h/2)(t-t_1) - i \int_{t_1}^t dt_2 v(t_2)}, \quad (7)$$

where the time-dependent applied voltage $v(t)$ denotes a rectangular pulse with duration time Δs and amplitude Δ . The time translation symmetry of $G_h^r(E_h, t, t_1)$ is destroyed by $v(t)$ (note that it is a typical n-i-n SET case). Comparing to $G_h^r(E_h, t, t_1)$, the time translation symmetry of $G_e^r(t, t_1)$ is destroyed by hole occupation number. Therefore, it is expected that $I_c(t)$ will be very different

from the tunnelling current of typical n-i-n SETs.^{10–12} The expression of base current arising from the electron-hole recombination of exciton state is given by

$$\begin{aligned}
J_b(t) & \\
&= e\alpha \int d\omega \omega^3 \int \frac{d\varepsilon}{\pi^2} f_e^<(\varepsilon - \omega/2) |A_e^a(\varepsilon - \omega/2, t)|^2 \\
&\times \Gamma_h f_h(\varepsilon - \omega/2) |A_h^r(\varepsilon - \omega/2, t)|^2
\end{aligned} \tag{8}$$

where $f_e^<(\epsilon) = (\Gamma_e f_e(\epsilon) + \Gamma_c f_c(\epsilon))$, $\alpha = 4n_r^3 \mu_r^2 / (6c^3 \hbar^3 \epsilon_0)$, where n_r and ϵ_0 are the refractive index and static dielectric constant of system, respectively. We see that $J_b(t)$ is determined by the time-dependent interband joint density of states and the factors of $f_e(\epsilon)f_h(\epsilon)$. To simplify the calculation of Eq. (8), we approximate it as

$$J_b(t) = eR_{eh}N_e(t)N_h(t), \tag{9}$$

where

$$N_e(t) = \int \frac{d\epsilon}{\pi} f_e^<(\epsilon) |A_e^r(\epsilon, t)|^2. \tag{10}$$

In Eq. (9) we define the time-independent spontaneous emission rate $R_{eh} = \alpha\Omega_{ex}^3$, where $\Omega_{ex} = E_g + E_e + E_h - U_{eh}$.

III. RESULTS AND DISCUSSION

Although tunnelling currents are more interesting than electron and hole occupation numbers from the experimental viewpoint, we still numerically solve Eqs. (6) and (10) and show the occupation number of electrons and holes as a function of time for different amplitudes of applied rectangular pulse voltage with $\Delta s = 3t_0$ at zero temperature in Fig. 4; the solid and dashed lines denote, respectively, the amplitude $\Delta = 30$ mV and $\Delta = 20$ mV. When holes are injected into the QD, the exciton resonant energy level for electrons is yielded. Consequently, the emitter supplies electrons into the QD via the exciton resonant energy level. In particular, some interesting oscillations superimpose on the charging and discharging processes of holes. Such a oscillatory behavior can not be observed for

electron tunnelling process. In addition, $N_e(t)$ exhibits a retarded response with respect to time. For $t = 1 t_0$, N_e is still less than 0.2 although N_h already reaches 0.8. This is because N_e is in proportion to $N_h^2(t)$ and the electron-channel behaviors as an opened system since $\Gamma_e = \Gamma_c = 0.5$ meV. On the other hand the hole channel behaviors as a closed system since $R_{eh} \ll \Gamma_h$.

Once electrons are injected into the QDs, the collector current and base current occur. Fig. 5(a) and (b) show the base current and collector current, respectively. When electrons tunnel into the QD from the emitter, photons are emitted from electron-hole recombination of exciton state, such photons should exhibit the antibunching feature with respect to time. Although in ref.[13] the antibunching feature of photons was reported, electrons and holes are injected into a single layer with dilute density of QDs. Therefore, in ref.[13] the tunnelling current arising from the spontaneous radiation of interband transition should be included the particle size distribution.¹⁴ It is worth noting that the photon number correlation function used to examine the antibunching characteristics are relevant with the base current behavior. Comparing to the base current, the exponential growth and decay of collector current are not so faster as that of base current. However, the collector current still mimics the behavior of base current. Due to the collector current in the units of $e \times meV/\hbar$, the current gain defined as $\beta = J_c/J_b$ can readily reach 100 for $\Gamma_e = \Gamma_c = 0.5$ meV. Consequently, the response of small signal can be amplified through the output of collector electrode. As mentioned, LESETs can be used as efficient SPS detectors.

Finally, Figs. 6(a) and (b) show the base current and collector current for two different tunnelling rates of hole. Other parameters used are the same as those employed in Figs. (4) and (5). Due to smaller tunnelling rate for solid line, hole occupation number is smaller in solid line than that of dashed line before pulse turns off. Subsequently, the electron occupation number also becomes smaller for $\Gamma_h = 0.5$ meV. Consequently, the base current is suppressed for $t \leq \Delta s = 3t_0$. However, the discharging time is enhanced for electrons and holes as $\Gamma_h = 0.5$ meV, the base current at $t \geq \Delta s = 3t_0$ is larger at $\Gamma_h = 0.5$ meV than $\Gamma_h = 1$ meV. The physical picture of collector current can be understood through the interpretation of the base current. The results shown in Figs. 5 and 6 indicate that the shape of collector current can be tuned by the combination of tunnelling rate

and duration time of applied signal. This could be used to manipulate the shape of time-dependent current.

IV. SUMMARY

We have studied the dynamic properties of LESETs made from a single QD embedded in a matrix connected with the emitter, collector and base electrodes. Electrons are transport carriers in the conduction band electrodes. As for valence band base electrode, holes created by light excitation or carrier doping play a role of switch trigger for the base current and collector current. The high current gain $\beta = J_c/J_b$ can be readily reached using semiconductor engineering fabrication technique. Such two outputs transistors exist potential application for the next generation optoelectronics.¹⁵

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Figure Captions

Fig. 1. The schematic band diagram for the single quantum dot (InAs) embedded in GaAs matrix connected with emitter, collector and base electrodes. The exciton energy level $E_{ex} = E_e - U_{eh}$ is 35 meV above the Fermi energy of the emitter and collector electrodes $E_{F,e} = 50$ meV. The resonant energy level of holes is 15 meV above the Fermi energy of base electrode $E_{F,h} = 50$ meV. The voltage difference between the emitter and collector is $V_{ec} = 45$ mV. A small ac signal $v(t)$ supplies holes into the quantum dot.

Fig. 2: U_e , U_{eh} , and U_h as functions of QD size for biexciton (solid), negative trion X^- (dotted) and positive trion X^+ (dashed). Note that the ratio $(h - 15\text{\AA})/(R_0 - 60\text{\AA}) = 1$ is used.

Fig. 3: Particle Coulomb interactions as functions of strength and direction of electric field for QD with radius of 7.5 nm and height of 3 nm. The solid line denotes the biexciton configuration, and dashed and dotted lines, respectively, denote a positive trion and a negative trion.

Fig. 4. Carrier occupation number as functions of time for two different amplitude of a rectangular pulse with duration time $\Delta s = 3 t_0$. Time is in units of $t_0 = \hbar/meV$. Temperature $k_B T = 0$ is considered throughout this study for simplicity.

Fig. 5. Diagram (a) and (b) correspond, respectively, for the base current and collector current. All parameters used are the same as those in Fig. 4. Base current and collector current are given in units of $J_0 = 2e R_{eh}$ and $e \times meV/\hbar$, where R_{eh} denotes the spontaneous emission rate.

Fig. 6. Diagram (a) and (b) correspond, respectively, for the base current and collector current. Solid line ($\Gamma_h = 0.5$ meV) and dashed line ($\Gamma_h = 1.0$ meV). $\Delta = 30$ mV. Other parameters used are the same as those in Fig. 5. Base current and collector current are given in units of $J_0 = 2e R_{eh}$ and $e \times meV/\hbar$, where R_{eh} denotes the spontaneous emission rate.















